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TECHNICAL NOTE

VERTICAL BEAMFORMER SIMULATION (U)

by

David Doan

Submitted to

U. S. Navy Electronics Laboratory
San Diego, California 92152
Attention: Code 1622C

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PROJECT NO. 002 009 16

VERTICAL BEAMFORMER SIMULATION

The performance of an all-digital beamformer for a 12-element line array has been investigated for several combinations of steering angle, frequency, and the number of bits carried at various points in the beamforming operation. The results are compared with a completely linear system, and with a linear system with discrete delay intervals and averaging between taps.

The digital system is shown in Fig. 1. Each hydrophone feeds a separate 4-bit quantizer. The quantizer characteristic is

$$
\begin{aligned}
Q(x) &= \mathrm{Int}\left(\frac{x}{a} + 8\right), \quad -7 \le \frac{x}{a} \le 7 \qquad (1)\\
&= 0, \quad \frac{x}{a} \le -7\\
&= 15, \quad \frac{x}{a} \ge 7
\end{aligned}
$$

Where a is the quantizer level spacing, and Int(x) = largest integer $\le$ x.

Each quantizer feeds a 4-bit delay line with taps spaced τ seconds apart. An N-position delay tap interpolator is employed to provide higher resolution for the delay intervals. This device outputs a linear average of the outputs of two adjacent taps. The output of the Nth tap, for an input X(t), is

$$
\mathrm{Int}\left\{\frac{X(t - k\tau)\ (N-n) + X\,[(t-(k+1)\tau]}{N'}\right\}, \qquad (2)
$$

providing an approximation to a true delay of $(k + \frac{n}{N})\,\tau$ seconds. Since the interpolation process multiplies the output of the quantizer by N and thus increases the number of bits to $4 + \log_2 N$, we divided by N' to truncate the output to the required value.

The outputs of the tap interpolators are weighted by suitable integer coefficients and summed to form the beam.

Two reference systems are employed to investigate the various sources of degradation in the digital system. One is a full linear system in which the quantizers have been removed and the delay lines are assumed to have infinite resolution so that tap interpolation is unnecessary. The other system is linear with respect to amplitude, but the delay lines have a resolution of τ seconds and the tap interpolation is retained.

In order to steer the beam to an angle β, we need a delay of

$$(j - 6.5)\,\frac{d}{c}\sin\beta$$

for the j^{th} element, where d is the distance between hydrophones, and c is the velocity of sound. The tap interpolator provides an approximation to a delay of $\left[k_j + (n_j/N)\right]\tau$. The values of k_j and n_j may be calculated for each element as follows:

for one half of the array, $7 \le j \le 12$

$$\begin{aligned} I_j &= \mathrm{Int}\left[(j - 6.5)\,\frac{Nd\sin}{c\tau}\,\beta + .5\right] \\ k_j &= \mathrm{Int}\left[\frac{I_j}{N}\right] \qquad (3) \\ n_j &= I_j - Nk_j \end{aligned}$$

The k_j and n_j for the other half of the array are found by symmetry:

for $1 \le j \le 6$

$$k_j = -k_{13-j} - 1$$

$$n_j = N - n_{13-j} \quad ,$$

Letting the signal be S(t), arriving from an angle α so that the signal appearing at the j^{th} hydrophone is

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$$S_j\left[t - (j - 6.5)\,\frac{d}{c}\sin\alpha\right],$$

we may write the output of the j^{th} tap interpolator as

$$T_j(t) = \mathrm{Int}\left\{\frac{(N-n_j)\,Q\left[S\left(t - (j - 6.5)\,\frac{d}{c}\sin\alpha + D_j\right)\right]}{N'} + \frac{n_j\,Q\left[S\left(t - (j - 6.5)\,\frac{d}{c}\sin\alpha + D_j + \tau\right)\right]}{N'}\right\} \qquad (4)$$

A similar expression describes the output of the j^{th} tap interpolator in an otherwise linear system:

$$C_j(t) = \frac{(N-n_j)\,S\left[t-(j-6.5)\frac{d}{c}\sin\alpha+D_j\right] + n_j S\left[t-(j-6.5)\frac{d}{c}\sin\alpha+D_j+\tau\right]}{N'a} \qquad (5)$$

The output from the j^{th} element of the full linear system is

$$L_j(t) = S\left[t - (j - 6.5)\left(\frac{d}{c}\sin\alpha - \frac{d}{c}\sin\beta\right)\right]\frac{N}{N'a}. \qquad (6)$$

The coefficients $\frac{1}{N'a}$ in Eq. (5) and $\frac{N}{N'a}$ in Eq. (6) are introduced to make the gain of the linear systems the same as that of the quantized system.

The system outputs are

$$y(t) = \mathrm{Int}\left[\frac{\sum_{j=1}^{12}\left(W_j\,T_j(t)\right) + \frac{M}{2}}{M}\right] \qquad (7)$$

for the quantized system,

$$x(t) = \frac{\sum_{j=1}^{12} W_j \; C_j(t)}{M} \tag{8}$$

for the linear system with tap interpolation; and

$$z(t) = \frac{\sum_{j=1}^{12} W_j \; L_j(t)}{M} \tag{9}$$

for the full linear system.

The W_j are integer shading coefficients and M is the round-off divisor to limit the output of the quantized system to a specified number of bits.

There are three sources of degradation in a system of this type:

1. Harmonic distortion due to quantization.
2. Distortion of the beam pattern because of the finite delay resolution.
3. Frequency distortion from the tap averaging.

The quantization introduces harmonic distortion which may be computed as follows:

Let ρ_{yz} be the normalized correlation coefficient between the output of the digital system and the output of the full linear system. Let σ_y^2 and σ_z^2 be the variance of the output of the quantized and linear systems, respectively. Then the total signal power at the output of the quantized system is $\sigma_y^2 \rho_{yz}^2$ and the total power output is σ_y^2. The power in the harmonics is $\sigma_y^2(1-\rho_{yz}^2)$. The total harmonic distortion is

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$$\% \text{ distortion} = \frac{\text{harmonic amplitude}}{\text{signal amplitude}} \times 100$$
$$= \sqrt{\frac{1 - \rho_{yz}^2}{\rho_{yz}}} \times 100. \tag{10}$$

We will not consider the degradation of the beam pattern in detail here. Instead, we shall assume that four point interpolation and a delay of 60 μ sec are adequate, and proceed to investigate the amplitude distortion caused by tap interpolation. The output of a tap interpolator in a linear system is

$$(N - n_j) \sin \omega t + n_j \sin \omega(t + \tau) = R \sin (\omega t + \varphi) \tag{11}$$

where

$$R = \sqrt{N^2 - 4n(N-n) \sin^2 \frac{\omega\tau}{2}} \quad ,$$

and

$$\varphi = \tan^{-1} \left[\frac{n \sin \omega\tau}{N - n + n \cos \omega\tau}\right] .$$

If $n = N/2$, then

$$R = N \cos \frac{\omega\tau}{2}, \text{ and}$$
$$\varphi = \frac{\omega\tau}{2} \quad , \tag{12}$$

so that the output is

$$N \sin \left[(t + \frac{\tau}{2})\omega\right] \cos \frac{\omega\tau}{2} . \tag{13}$$

The delay from this interpolation is exactly half way between the taps on the delay line and independent of frequency. The amplitude is decreased with increasing frequency by $\cos \frac{\omega\tau}{2}$.

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For $N = 4$, $n = 1$ or 3 we find for the amplitude function

$$R = 2\sqrt{4 - 3 \sin^2 \frac{\omega\tau}{2}} \ . \tag{14}$$

For the delay, with $n = 1$

$$\varphi = \tan^{-1}\left\{\frac{\sin \omega\tau}{3 + \cos \omega\tau}\right\} , \tag{15}$$

and for $n = 3$

$$\varphi = \tan^{-1}\left\{\frac{3 \sin \omega\tau}{1 + 3 \cos \omega\tau}\right\} . \tag{16}$$

For $\tau = 60\,\mu$sec delays are shown in Table I, and the amplitudes, in dB relative to the ideal, are shown in Table II.

TABLE I

n \ f	2 kHz	3 kHz	Ideal
0	0	0	0
1	14.44	13.7	15
2	30	30	30
3	45.56	46.3	45
4	60	60	60

TABLE II

n \ f	2 kHz	3 kHz
0	0	0
1	-0.47	-1.05
2	-0.64	-1.48
3	-0.47	-1.05
4	0	0

The variations in effective time delay and shading with frequency will introduce variations of the beam pattern with frequency in addition to those caused by the changing ratio of wavelength to array dimensions.

The equations for the output of the various systems have been programmed for evaluation by a computer, using a sine wave signal. The program calculated the correlation coefficients and relative gains of the digital system vs. the full linear system and vs. the linear system with tap interpolation.

The results are summarized in Table III, and plots of the output of the digital system vs. the full linear system are shown in Figs. 2 through 14.

DISCUSSION OF RESULTS

The output distortion depends on the steering angle, the frequency, the number of bits in the tap interpolator, and the shading function. If we use total harmonic distortion as a criterion, the worst cases with respect to steering angle are those which require the same tap on the tap interpolators for each element. For instance, at 0° (perpendicular to the column) the output of each element is taken directly from a delay line tap. At 26.8°, each output is from the center tap of the tap interpolator. These are worst cases since all of the quantizers step at the same time, producing a transfer function identical to that of the basic quantizer. For other steering angles, the output is the sum of signals taken from various taps on the interpolators. This, together with the residual phase errors, tends to increase the effective amplitude resolution of the system and reduce the distortion. The relative phase error increases with frequency so that, within limits, the distortion is reduced with increasing frequency.

When the beam is steered to 26.8° from broadside, the output of each element is taken from the middle tap of the tap interpolator. The transfer function becomes that of one quantizer followed by the tap interpolator. This transfer function does not have a step at the origin. This may lead to difficulties with weak signal suppression and needs to be investigated further.

The four point tap interpolator operating on the output of a four bit quantizer requires six bits to maintain the full accuracy of the system. Truncating the output to four bits increases the distortion. However, only in the case of steering to 26.8^{0} does the distortion rise above that obtained at broadside. Truncation does destroy the symmetry of the transfer function. The quantizer itself will produce only odd harmonics. The truncated output of the tap interpolator will contain even harmonics as well.

It should be noted that the distortion is also a function of the sum of the weights used for shading. This number affects the way in which the final output is resolved after round-off. For the particular case considered here, the sum of the weight is 72. The amplitude of one output step at broadside before round-off is 72 x 4 = 288. The maximum output is 72 x 4 x 15 = 4320. If we truncate this to approximately 7 bits by dividing by 32, we find that the maximum output is 135 and each step is 9. This is resolved exactly at the final output. However, the maximum output is slightly greater than can be handled by 7 bits.

If we reduce the sum of the weights to 68, each step will be 68 x 4/32 = 8.5 and the maximum output is 127.5. Thus, we find that the broadside output has alternate steps of 8 and 9 units, but the output is within the range of 7 bits. The distortion is increased from 6.00% to 6.07% by the change in shading pattern.

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TABLE III

SUMMARY OF RESULTS FOR 12 ELEMENTS WEIGHTED 2, 3, 5, 7, 9, 10, 10, 9, 7, 5, 3, 2
ELEMENT SPACING = 133 msec, TAP SPACING = 60 msec
4 BIT QUANTIZER, SIGNAL AMPLITUDE = 7 x QUANTIZER STEP

Steering Angle Degrees	Freq. kHz	Tap Interpolator Bits	P_{yz}	Distortion %	Gain re TI Linear, dB	Gain re full Linear, dB	Transfer Plot Fig.
0 (broadside)	3	6	0.99820644	6.00	-0.16	-0.16	2
8	3	6	0.99986494	1.64	-0.17	-1.23	3
8	3	4	0.99967727	2.54	-0.15	-1.21	4
8	2	6	0.99982087	1.89	-0.17	-0.63	5
8	2	4	0.99966083	2.60	-0.20	-0.67	6
20	3	6	0.99985822	1.68	-0.18	-1.22	7
20	3	4	0.99968003	2.53	-0.14	-1.18	8
20	2	6	0.99973547	2.30	-0.16	-0.64	9
20	2	4	0.99954186	3.03	-0.20	-0.66	10
26.8	3	6	0.99894267	4.60	-0.16	-1.63	11
26.8	3	4	0.99721507	7.48	-0.15	-1.62	12
26.8	2	6	0.99893978	4.61	-0.16	-0.80	13
26.8	2	4	0.99748702	7.10	-0.15	-0.78	14

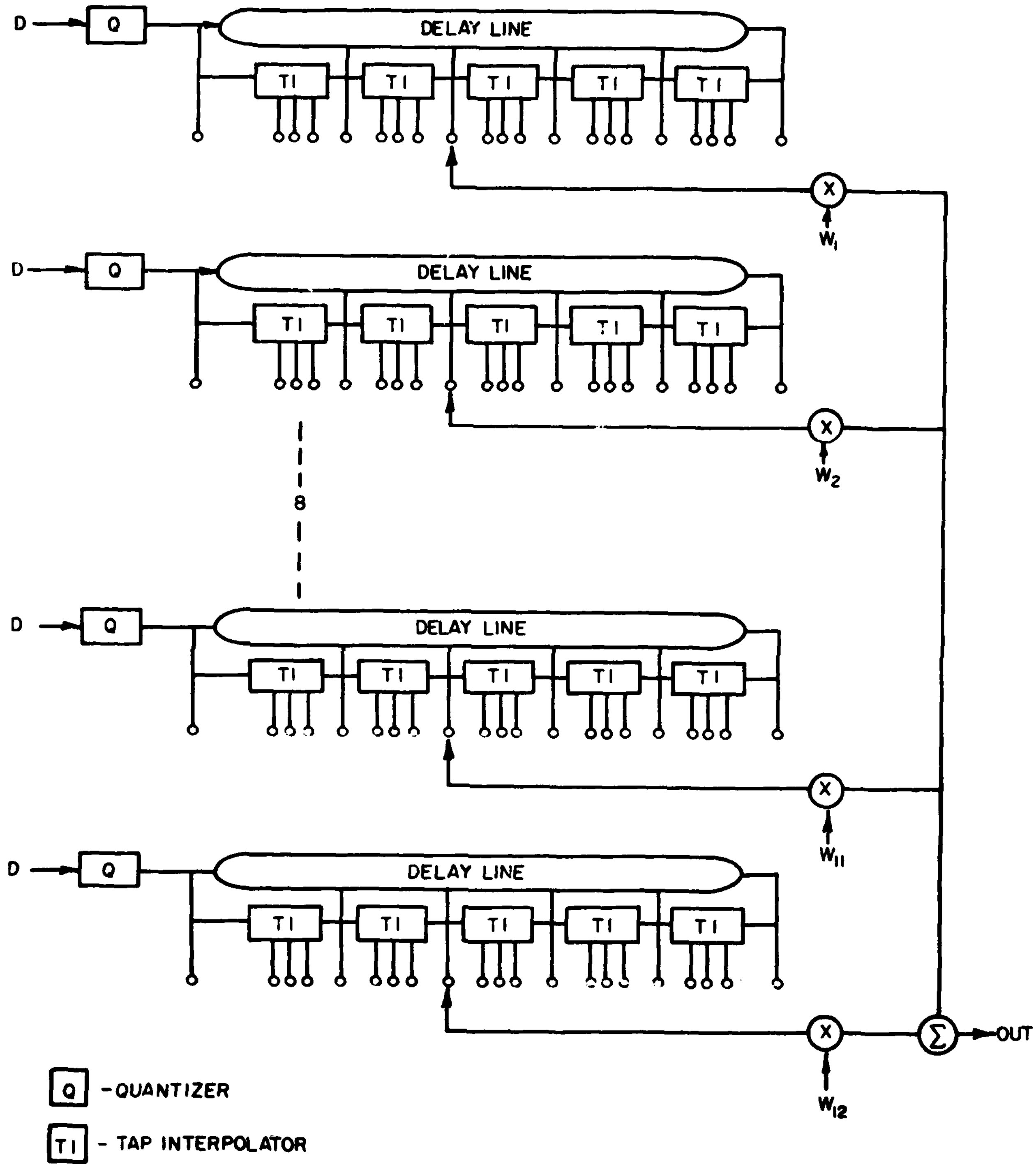

FIG. I — BLOCK DIAGRAM OF BEAMFORMER

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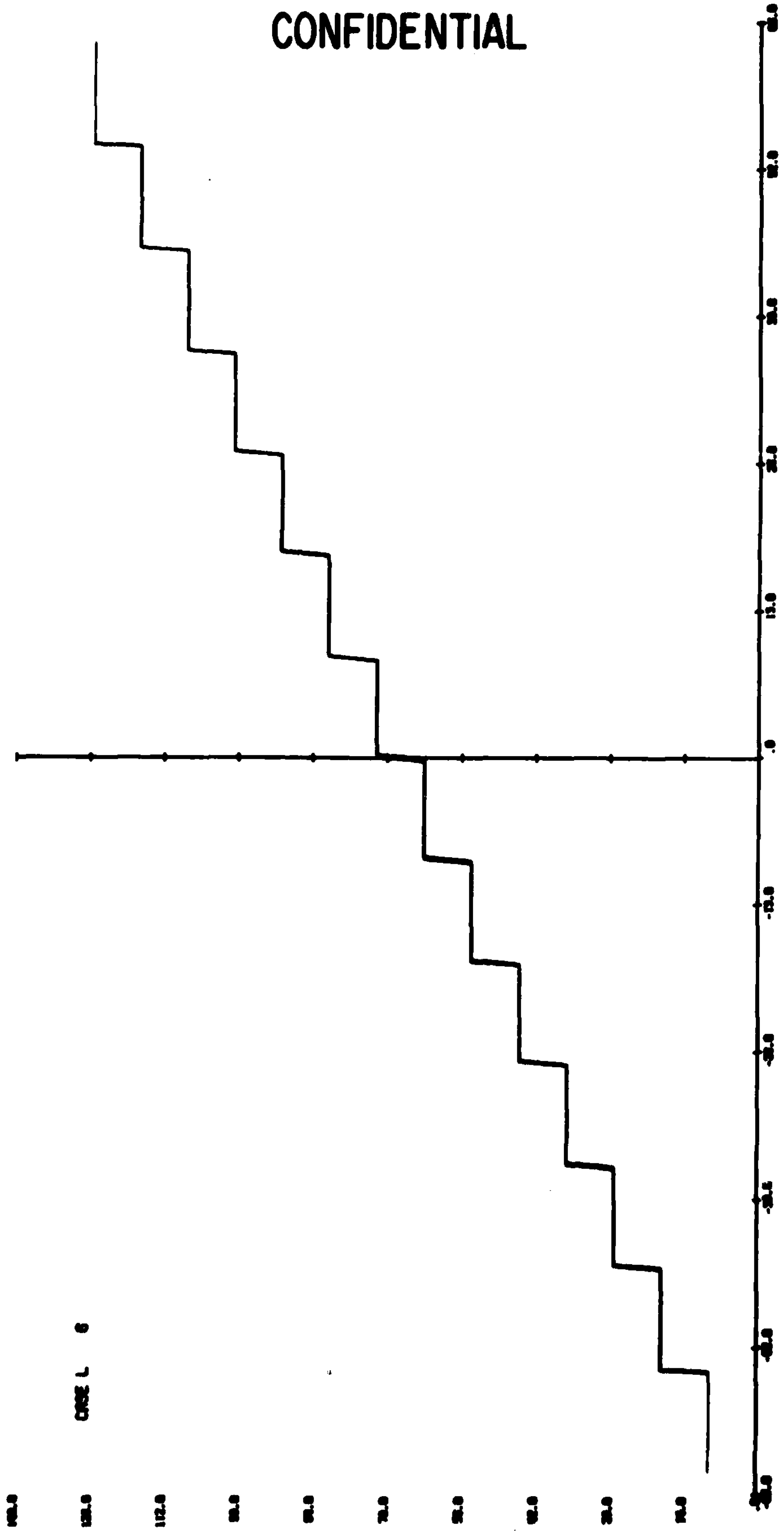

Fig. 2 - Transfer Function at Broad~~cast~~side

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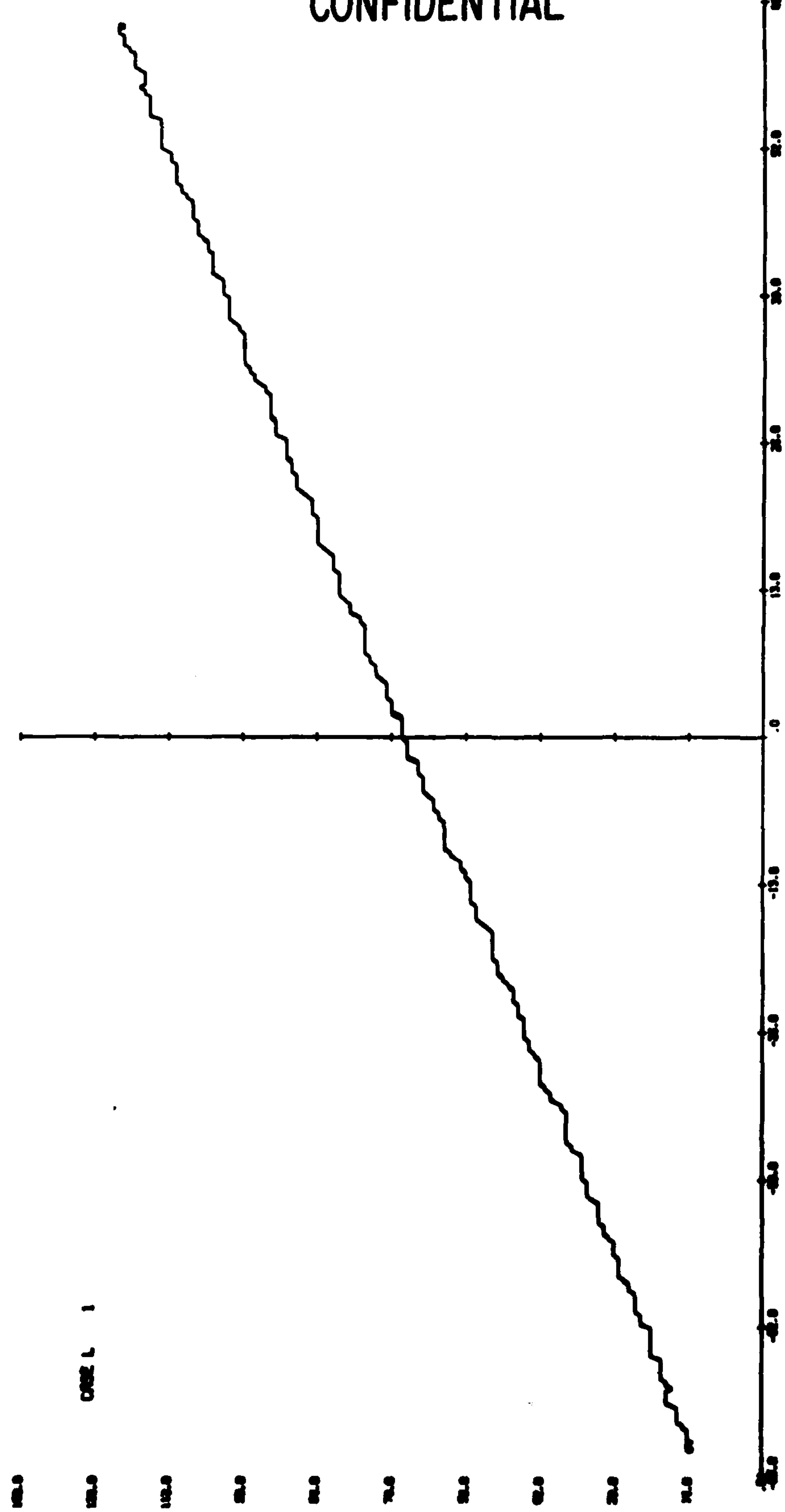

Fig. 3 - Transfer Function for Beam Steered to 8° at 3 kHz and 6 bit Tap Interpolation

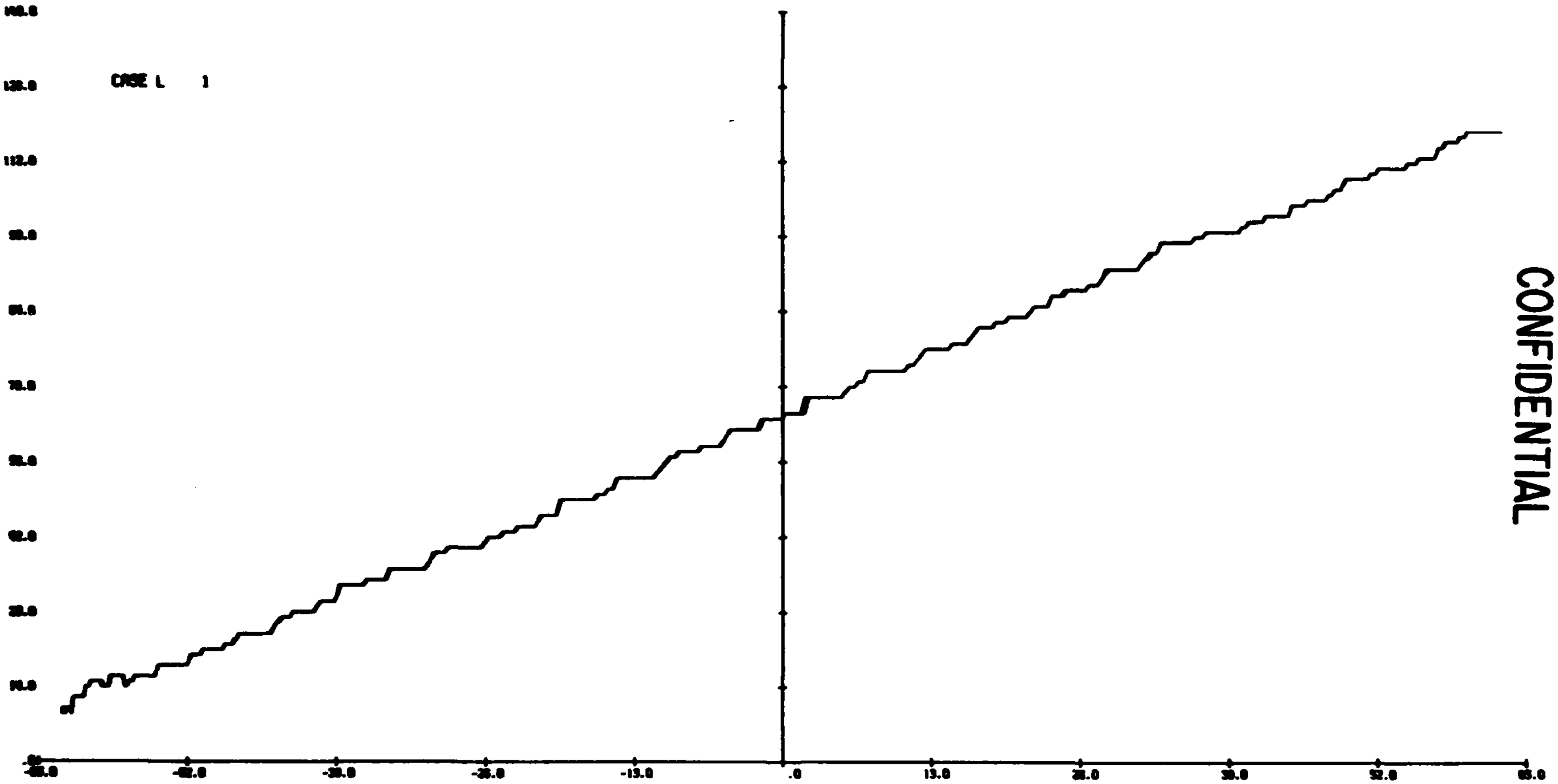

Fig. 4 - Transfer Function for Beam Steered to 8° at 3 kHz and 4 bit Tap Interpolation

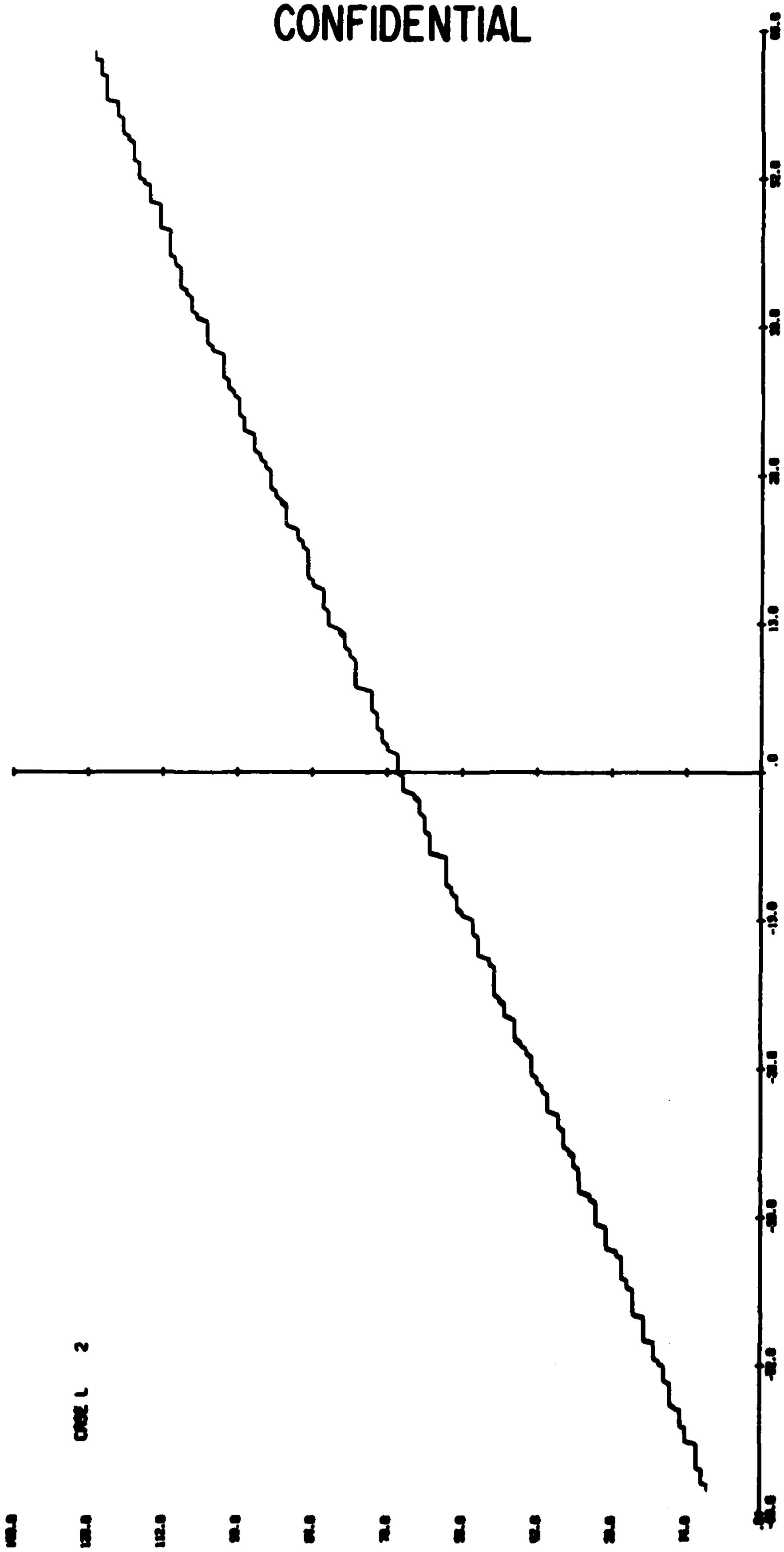

Fig. 5 - Transfer Function for Beam Steered to 8^{o} at 2 kHz and 6 bit Tap Interpolation

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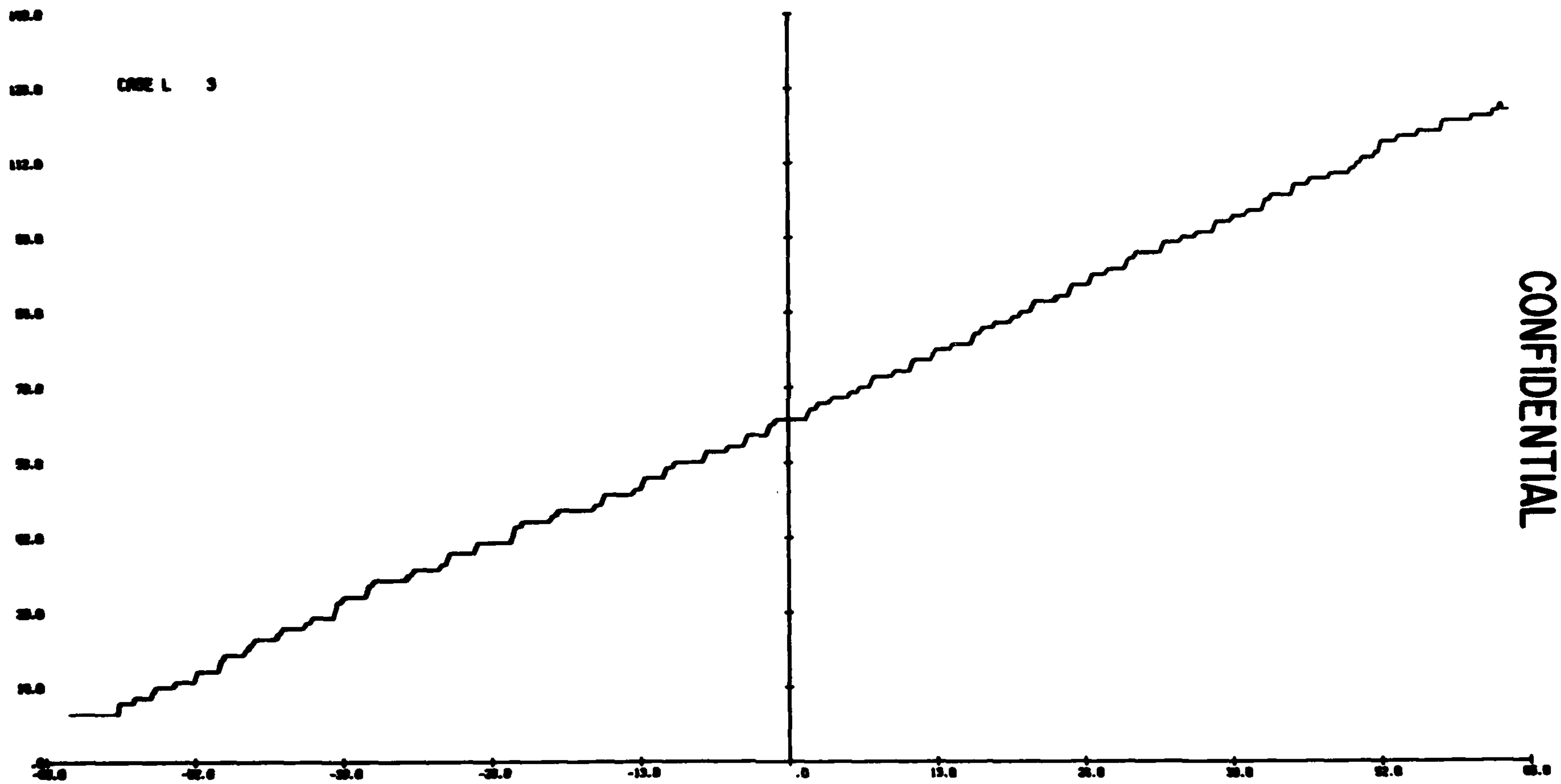

Fig. 6 - Transfer Function for Beam Steered to 8° at 2 kHz and 4 bit Tap Interpolation

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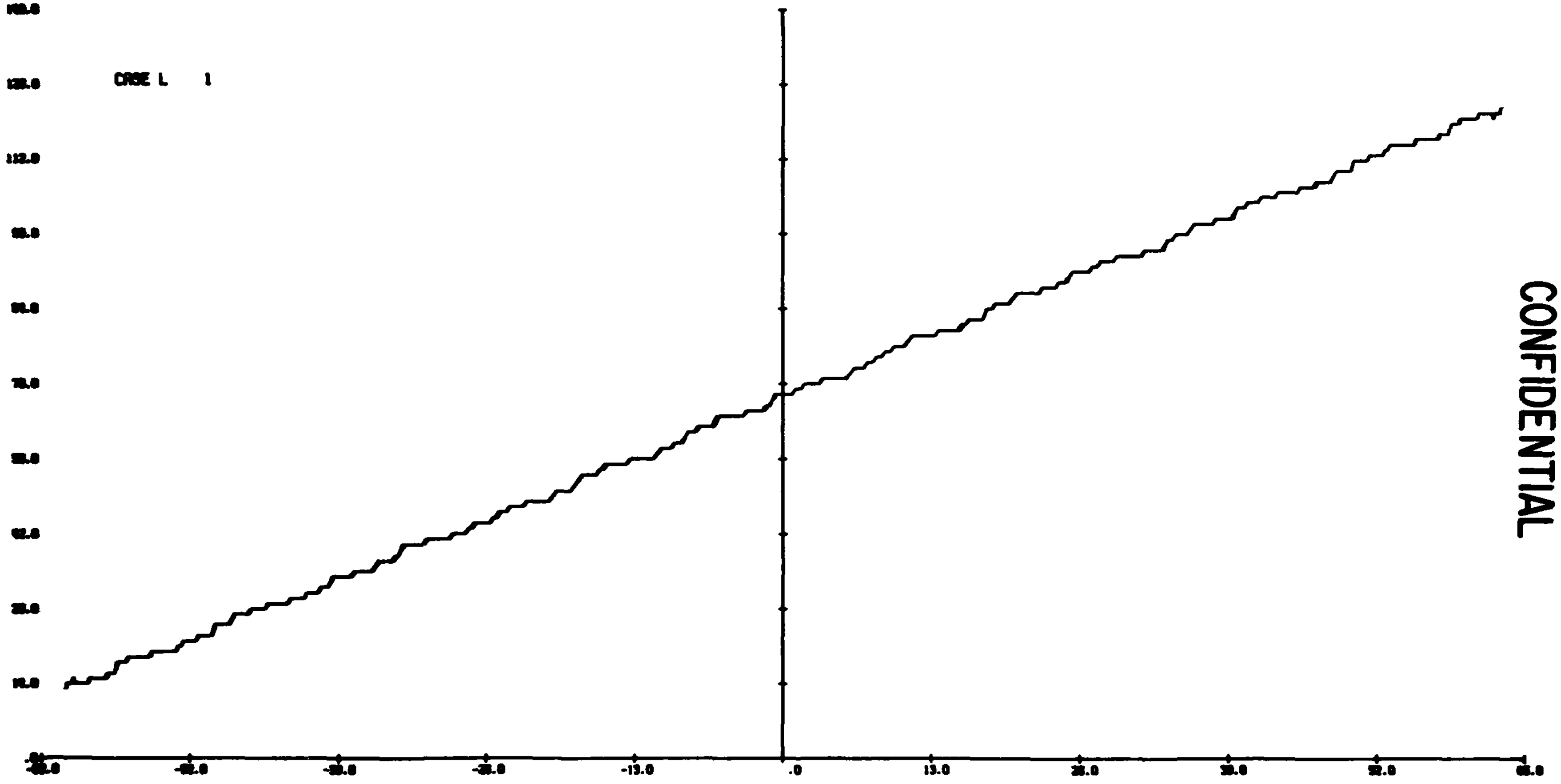

Fig. 7 - Transfer Function for Beam Steered to 20° at 3 kHz and 6 bit Tap Interpolation

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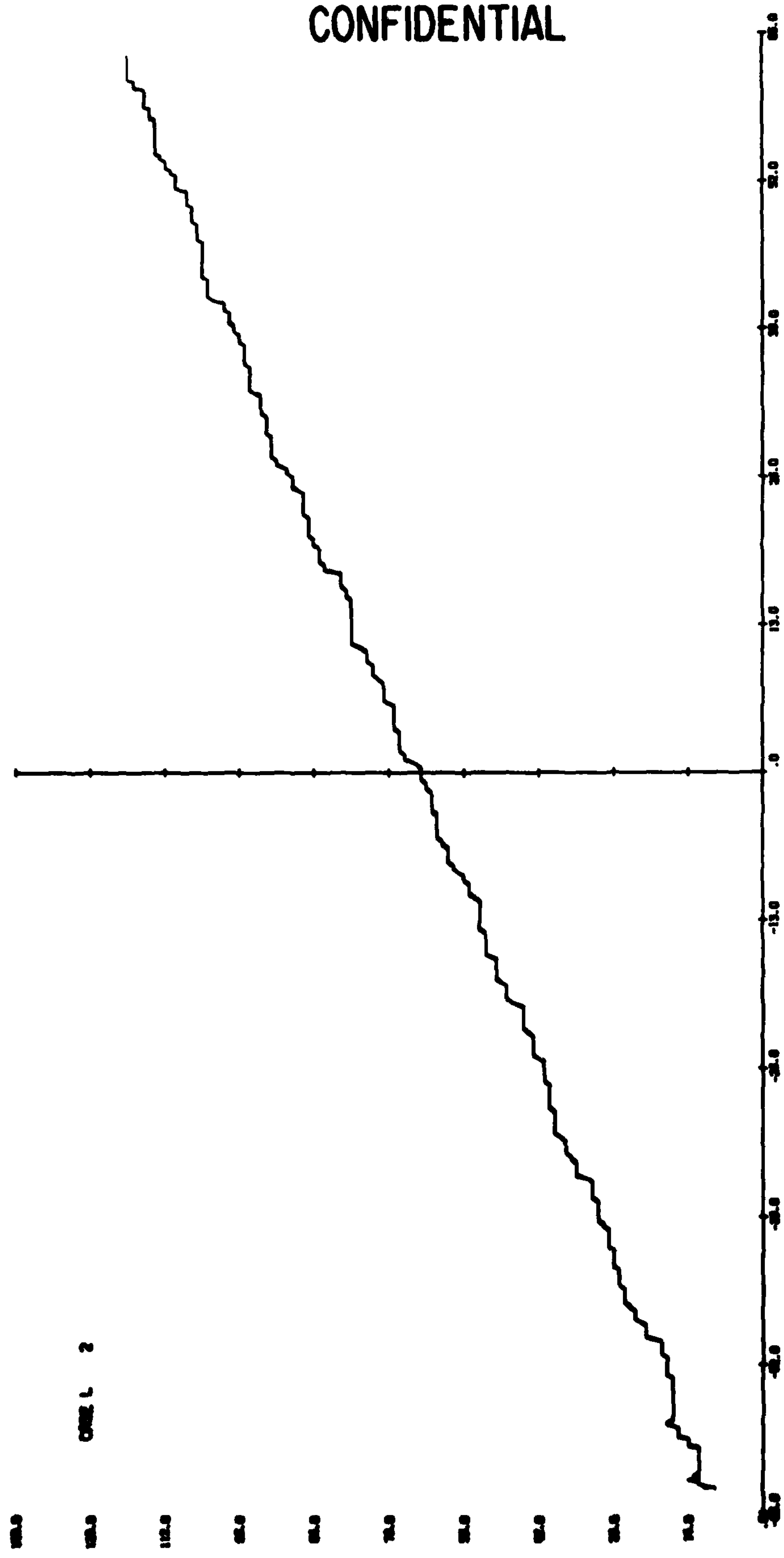

Fig. 8 - Transfer Function for Beam Steered to 20^{o} at 3 kHz and 4 bit Tap Interpolation

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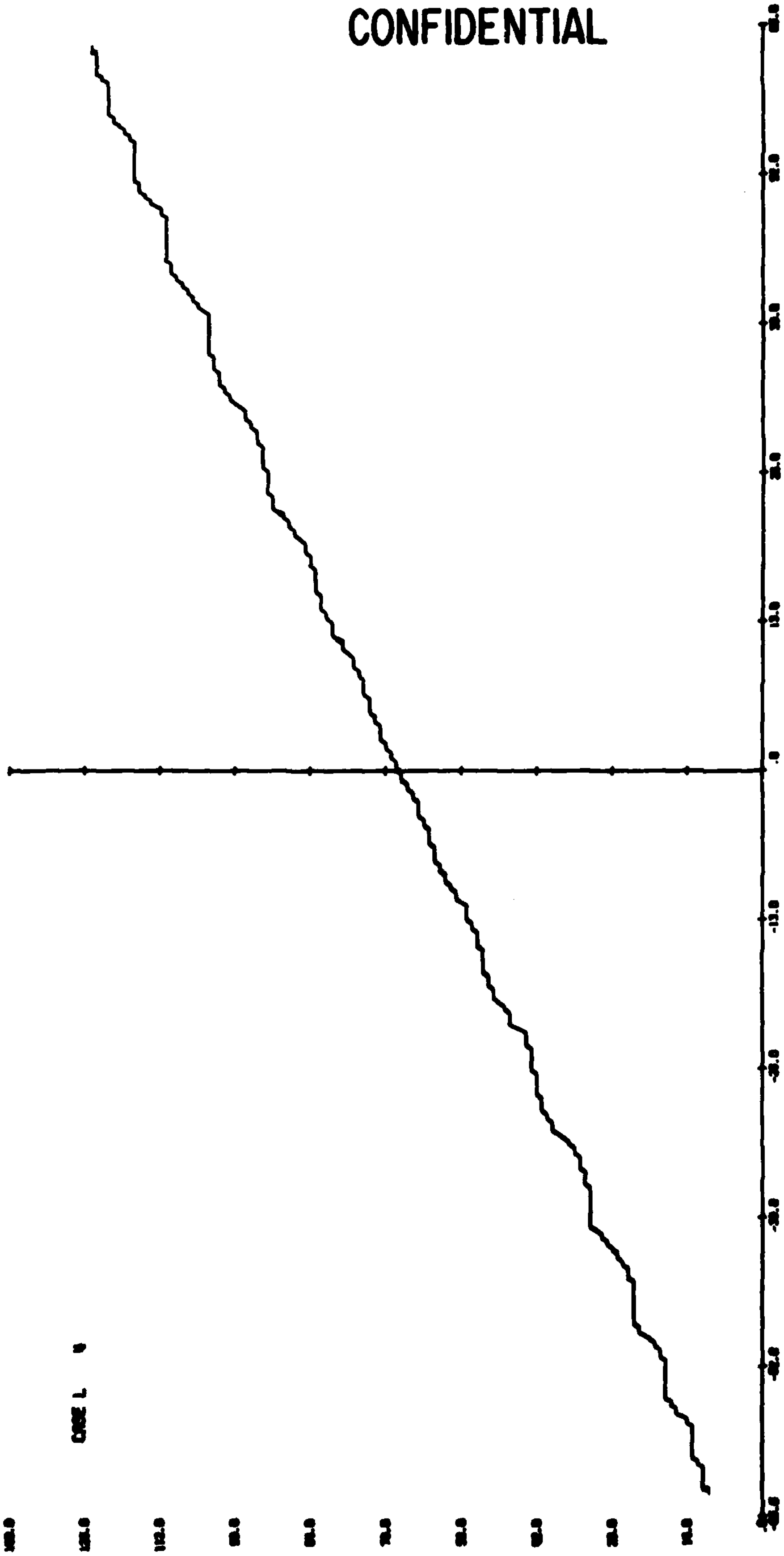

Fig. 9 - Transfer Function for Beam Steered to 20° at 2 kHz and 6 bit Tap Interpolation

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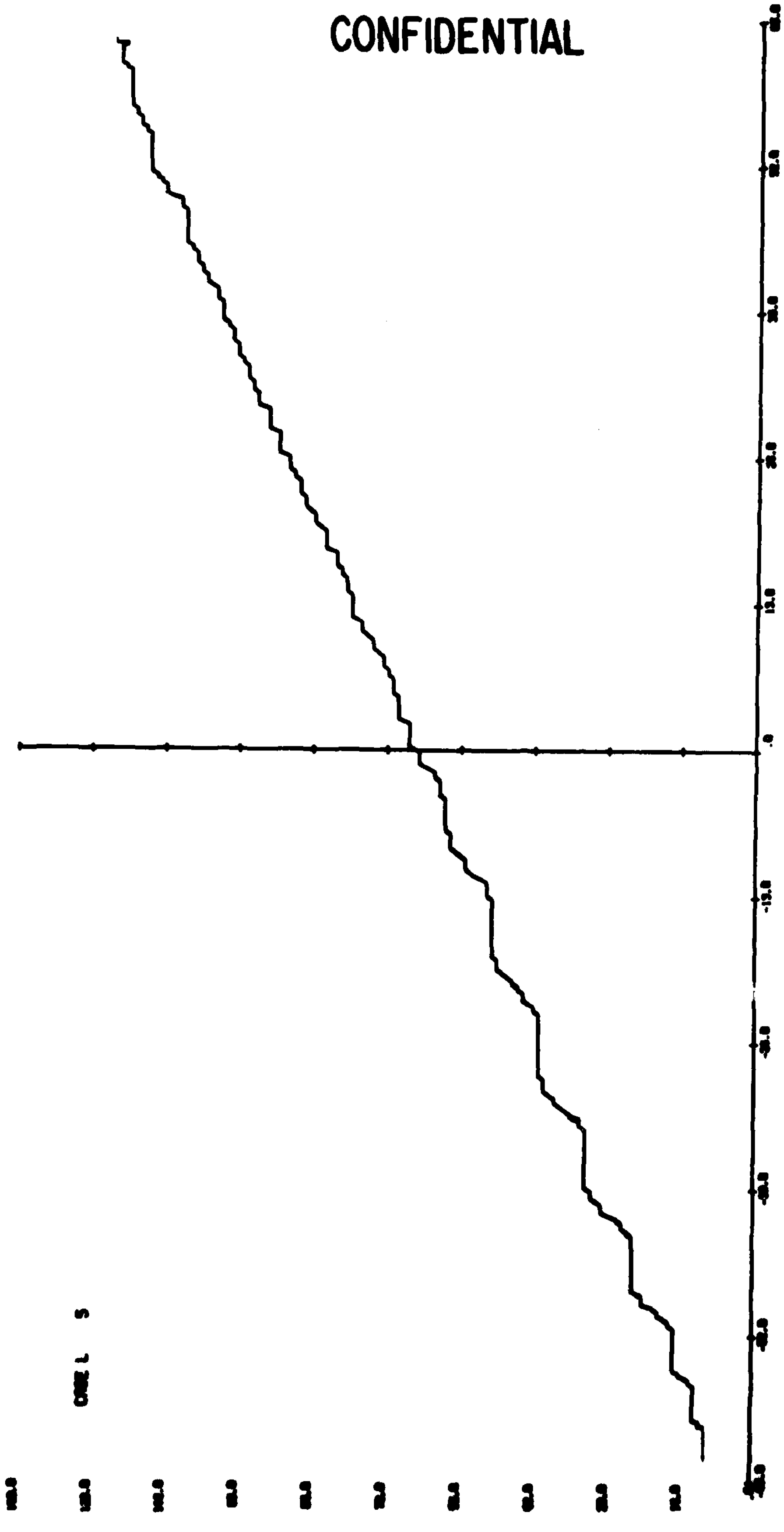

Fig. 10 - Transfer Function for Beam Steered to 20^{o} at 2 kHz and 4 bit Tap Interpolation

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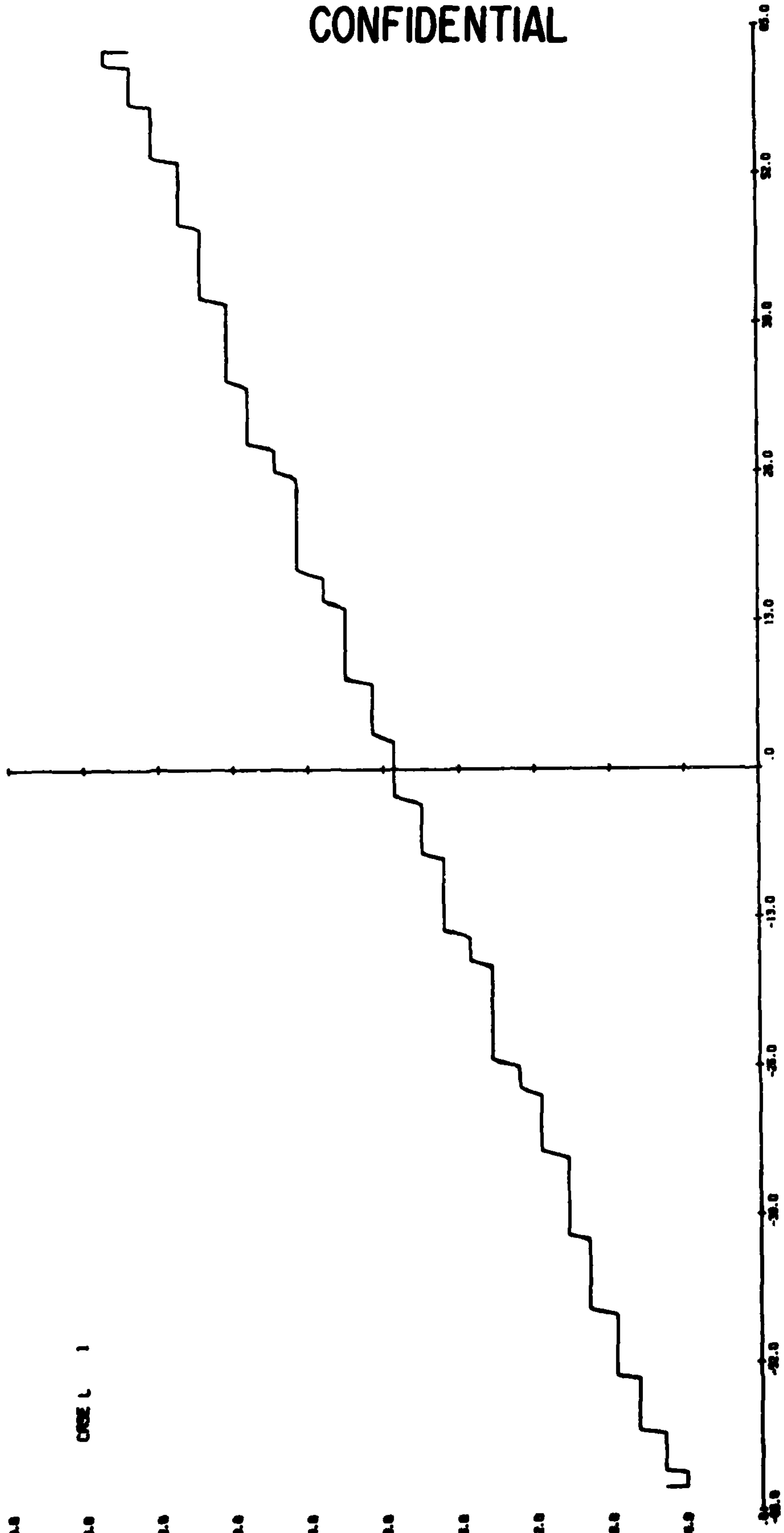

Fig. 11 - Transfer Function for Beam Steered to 26.8° at 3 kHz and 6 bit Tap Interpolation

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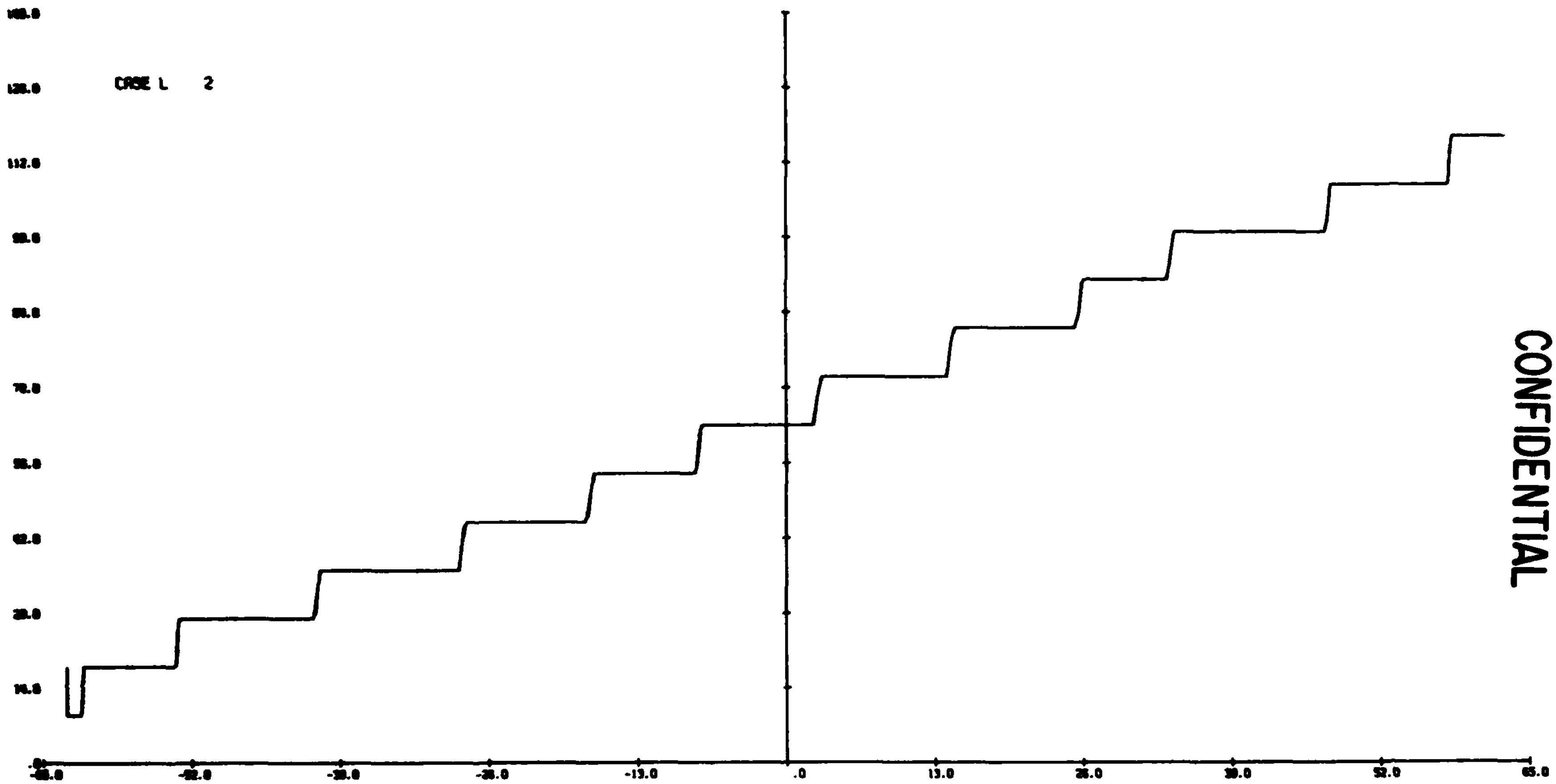

Fig. 12 - Transfer Function for Beam Steered to 26.8° at 3 kHz and 4 bit Tap Interpolation

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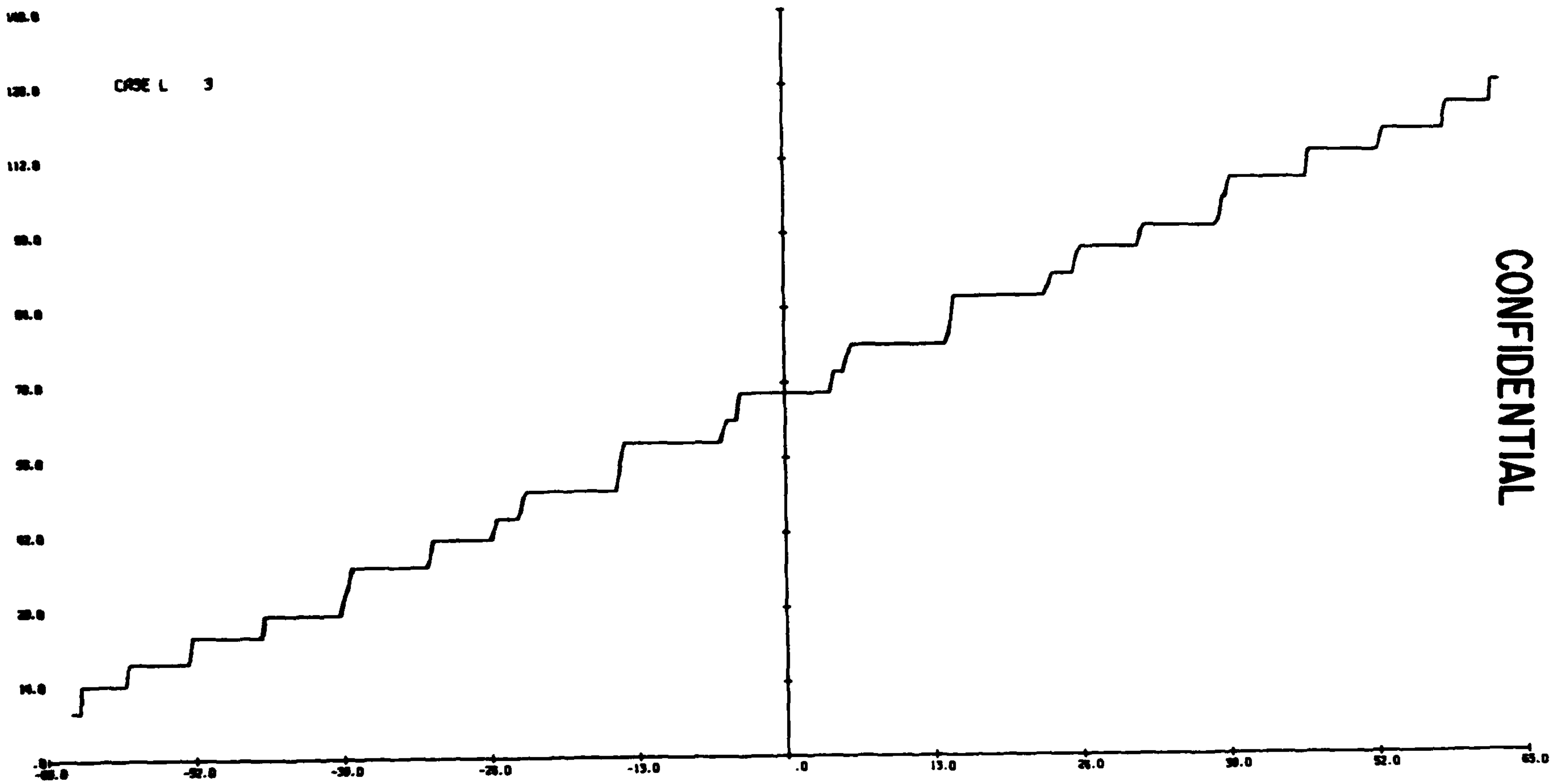

Fig. 13 - Transfer Function for Beam Steered to 26.8° at 2 kHz and 6 bit Tap Interpolation

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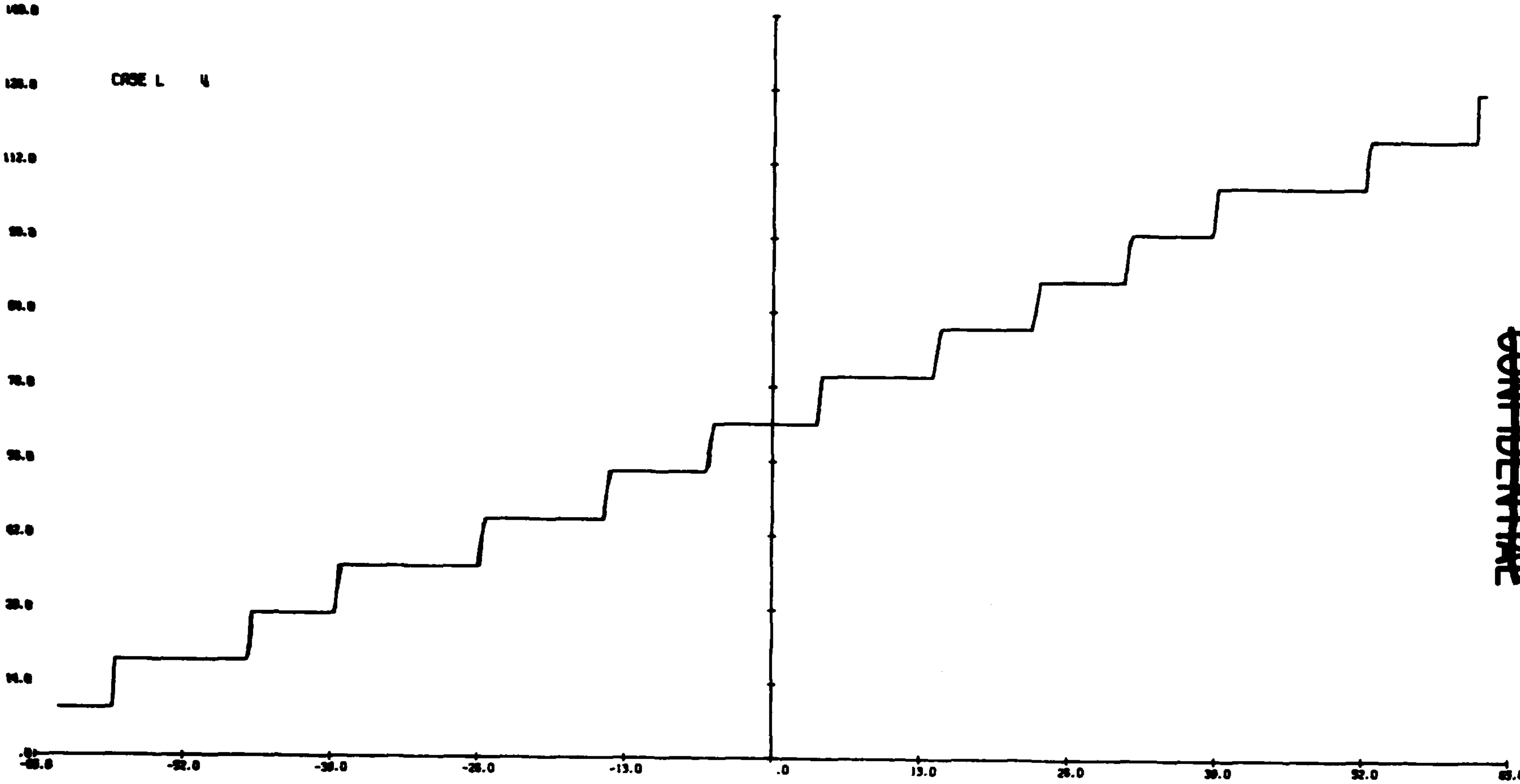

Fig. 14 - Transfer Function for Beam Steered to 26.8° at 2 kHz and 4 bit Tap Interpolation

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